MA-128 **Review for Final Exam** **Fall 2025**

1. The position of an object at time t is given by $S(t) = 2t^3 - 13t^2 + 24t + 100$.
   (a) Find the velocity and the acceleration of the particle at t = 1.
   (b) Find the times at which the particle comes to a rest.

2. Given the function $f(x) = x^2 + 12x$.
   (a) Find the average rate of change as x changes from 3 to 5.
   (b) Find the instantaneous rate of change of the function when $x = 3$.

3. Find y' if $y = x\ln(4x^2 + 1)$.

4. Find the derivative of $y = \frac{\sin x}{3+2\cos x}$

5. Find $\lim\limits_{x\to 9} \frac{\sqrt{x}-3}{x-9}$ and simplify.

6. Find $\lim\limits_{x\to -1} \frac{x^2+4x+3}{x^2-5x-6}$

7. If $f(x) = 3x^7 - \frac{4}{x^2} + 4\sqrt[3]{x}+1$, find $f'(x)$ and $f''(x)$.

8. Evaluate $\int \frac{x}{x^2+1}\,dx$

9. Find y'if $y = 3(x^3-7x+2)^{15} + 4x^2 - 2$

10. Find y'if $y = 2x + \sqrt{x^4 - 9x + 3}$

11. Find the area bounded by the curves $y = 8x - x^2$ and $y = 3x$.

12. Find the area between: $y = 2x + 3$ and $y = -x^2 + 10x - 9$.

13. Let $y = 4x^2 - 4x + 3$.
   (a) Find an equation for the tangent line to this graph at $x = 2$.
   (b) Find the point(s) on this graph at which the tangent line is horizontal.

14. Let $f(x) = x^3 + 3x^2 - 9x - 13$.
   (a) Find the interval(s) where the function is increasing and the interval(s) where the function is decreasing.
   (b) Find the local maximum and local minimum points.
   (c) Find the inflection point(s).
   (d) Find the interval(s) where the graph of the function is concave up and the interval(s) where the graph is concave down.
   (e) Graph the function.

15. Solve the following differential equations:
   (a) $y' = 4x^3 + 2x + 5$ where $y = 10$ when $x = 1$.
   (b) $\frac{dy}{dx} = 3x + xy$ where $y = 2$ when $x = 0$.

16. The total cost of producing x shirts is: $C(x) = 1000 + 20\sqrt{x}$, in dollars.
   (a) Find the average cost per shirt when x units are produced.
   (b) Find the rate at which this average cost is changing when 400 shirts are being produced.

17. Find the derivative of each of the following:
   (a) $y = 20\sin 15x$

   (b) $y = \tan^3 x$

   (c) $y = e^{13x}\cos x$

   (d) $f(x) = xe^{-x^2}$

18. Evaluate the following:

   (a) $\int \cos 8x\, dx$

   (b) $\int_0^{\frac{\pi}{2}} \sin^{10} x \cos x\, dx$

   (c) $\int_0^{\frac{\pi}{3}} \sec^2 x\, dx$

   (d) $\int e^{2x} dx$

19. A rectangular box with a volume of 32 cubic feet is to be constructed with a square base and a top and bottom. The cost per square foot for the top and bottom is 15 cents, and for the sides it is 10 cents. Find the dimensions that will minimize the cost of the box. Round to two decimal places.

20. The power available on a satellite, in watts, is given by $P = 100e^{-0.005t}$, where t is the time in days since the satellite was deployed.
   (a) How much power is available after 100 days? Round to two decimal places.
   (b) The satellite cannot operate on less than 10 W of power. How long can the satellite operates? Round to two decimal places.

21. If $f(x) = \frac{3x}{x^2+9}$, find $f'(x)$ and $f'(1)$.

22. If $g(x) = x\sqrt{x^2+5}$, find $g'(x)$ and $g'(2)$.

23. Solve the differential equation: $\frac{dy}{dx} = 2y - 10$ if $y = 15$ when $x = 0$.

Answers:

1. (a) Velocity = 4 and acceleration = -14

   (b) At $t = \frac{4}{3}$ and $t = 3$

2. (a) 20 (b) 18
3. $y' = \ln(4x^2 + 1) + \frac{8x^2}{4x^2+1}$
4. $y' = \frac{3\cos x + 2}{(3+2\cos x)^2}$
5. $\frac{1}{6}$
6. $-\frac{2}{7}$
7. $f'(x) = 21x^6 + \frac{8}{x^3} + \frac{4}{3x^{\frac{2}{3}}}$ and $f''(x) = 126x^5 - \frac{24}{x^4} - \frac{8}{9x^{\frac{5}{3}}}$
8. $\frac{1}{2}\ln(x^2 + 1) + C$
9. $y' = 45(x^3 - 7x + 2)^{14}(3x^2 - 7) + 8x$
10. $y' = 2 + \frac{4x^3 - 9}{2\sqrt{x^4 - 9x + 3}}$
11. $\int_0^5 (8x - x^2 - 3x)\ dx = \frac{125}{6}$
12. $\int_2^6 \left((-x^2 + 10x - 9) - (2x + 3)\right)dx = \frac{32}{3}$
13. (a) $y = 12x - 13$

    (b) At the point $(\frac{1}{2}, 2)$
14. (a) Increasing on $(-\infty, -3)$ and $(1, \infty)$

    Decreasing on (-3, 1)

    (b) Maximum at $(-3, 14)$, minimum at $(1, -18)$

    (c) $(-1, -2)$

    (d) Concave up on $(-1, \infty)$ and concave down on $(-\infty, -1)$
15. (a) $y = x^4 + x^2 + 5x + 3$ (b) $y = 5e^{\frac{x^2}{2}} - 3$
16. (a) $AC(x) = \frac{1000}{x} + \frac{20}{\sqrt{x}}$

    (b) $AC'(x) = -\frac{1000}{x^2} - \frac{10}{x^{\frac{3}{2}}}$ and $AC'(400) = -\frac{3}{400}$ dollars per shirt
17. (a) $y' = 300\cos 15x$ (b) $y' = 3\tan^2 x\ \sec^2 x$

    c) $y' = 13e^{13x}\cos x - e^{13x}\sin x$ (d) $f'(x) = e^{-x^2} - 2x^2e^{-x^2}$
18. (a) $\frac{\sin 8x}{8} + C$ (b) $\frac{1}{11}$ (c) $\sqrt{3}$ (d) $\frac{e^{2x}}{2} + C$

19. The box should have a height of 4.16 feet, and the base should be a square with sides equal to 2.77 feet.
20. (a) 60.65 watts (b) 460.52 days
21. $f'(x) = \frac{27 - 3x^2}{(x^2+9)^2}$ and $f'(1) = \frac{6}{25}$
22. $g'(x) = \sqrt{x^2 + 5} + \frac{x^2}{\sqrt{x^2+5}}$ and $g'(2) = \frac{13}{3}$
23. $y = 5 + 10e^{2x}$